# Graphs and Algorithms

## Exercise 1 (Small and regular)

Find a 3-regular simple graph with connectivity 1. Is your example minimal?

#### Exercise 2 (Long cycles)

Show that for  $k \ge 2$  every k-connected graph with at least 2k vertices contains a cycle of length at least 2k. Describe a k-connected graph with at least 2kvertices that does not contain a cycle of length 2k + 1.

#### Exercise 3 (Connectivity of the *k*-dimensional Cube)

The k-dimensional cube graph  $Q_k$  is the graph on the vertex set  $\{0,1\}^k$  in which two vertices are joined by an edge if and only if they differ in exactly one coordinate. What is the connectivity  $\kappa(Q_k)$  of this graph?

## Exercise 4 (Subgraph with large minimum degree)

Let G = (V, E) be a graph on n vertices with at least  $d \cdot n^2$  edges, where  $d \in (0, 1)$ . Prove that

- 1. there exists a subset  $V' \subseteq V$  of size at least  $n\sqrt{d}$ , such that  $\delta(G[V']) \geq \frac{dn}{2}$ ,
- 2. there exists a subset  $V' \subseteq V$  such that  $\delta(G[V']) \ge dn$ .

Note: parts (1) and (2) represent a tradeoff between the size of the induced subgraph and its minimum degree.

DISCUSSION OF THE SOLUTION IN THE EXERCISE CLASS ON 14.3.2013.