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Exercise sheet 5

Graphs and Algorithms

Exercise 1 (Matchings in trees)

Prove that every tree admits *at most* one perfect matching.

Exercise 2 (Line Graph of Regular Graphs)

Let G be a connected graph which is regular of degree $r \geq 2$. Prove that the line graph $L(G)$ is Eulerian.

Exercise 3 (Knights on the Chessboard)

What is the maximum number of knights that can be placed on a chessboard such that no two threaten each other?

Hint: Formulate the problem in the language of graph theory and exploit the relations between independent sets, vertex covers and matchings. Fun fact: you can start with a knight on any square and jump around visiting each square exactly once.

Exercise 4 (Fast Matching Approximation - Exam 2012)

Let G be a bipartite graph on n vertices and m edges, in which the size k of a maximum matching satisfies $k \geq n/4$. Modify the Hopcroft-Karp algorithm to find a matching of size at least $0.99 \cdot k$ in time $O(m)$.

Discussion of the solution in the exercise class on 4.4.2013.