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Exercise sheet 7

Graphs and Algorithms

Exercise 1 (Greedy is good, greedy is bad)

(a) Show that for every graph G there exists a permutation of the vertices such that Greedy uses only $\chi(G)$ many colors.

(b) Show that there is a tree $T = (V, E)$ on n vertices and a permutation $\pi : \{1, 2, \dots, n\} \to V$ such that the algorithm Greedy-Coloring(T, π) needs $\Omega(\log n)$ colors.

Exercise 2 (Directed edge coloring)

Let G be a directed graph such that all vertices have in- and out-degree both at most k. Prove that you can color the edges of G with at most k colors such that for each vertex of G all out-edges have different colors and all in-edges have different colors.

Exercise 3 (Girth six, $\chi(G) = k$)

Let $k \geq 2$ and n be integers and G a graph on n vertices with girth 6 and $\chi(G) = k$. Prove that the following construction creates a graph H with girth 6 and $\chi(H) = k + 1$. Create a set T of nk vertices. Then for every subset S of T with cardinality n, connect a *new* copy of G to S by an arbitrary perfect matching between the vertices in S and those of the copy of G.

In-class exercise on 02.05.2012.