Graphs and Algorithms

Exercise 1 (Petersen, minors and subdivisions)

Consider the so called Petersen graph in the picture below. Prove that the Petersen graph...

- a) ... contains a subdivision of $K_{3,3}$.
- b) ... contains no subdivision of K_5 .
- c) ... contains both $K_{3,3}$ and K_5 as minors.



Exercise 2 (Outerplanar graphs)

We say that a graph G is *outerplanar* if there exists a plane drawing of G such that every vertex belongs to the outerface.

- (a) Give a polynomial time algorithm which checks whether a given graph G is outerplanar.
- (b) Prove that every outerplanar graph is 3-colorable.
- (c) Use part (b) to prove the art gallery problem: If an art gallery is laid out as a simple polygon with n sides (see Figure 1 for an example), then it is possible to place $\lfloor n/3 \rfloor$ guards on the boundary or the interior of the polygon such that every point of the interior is visible to some guard.
- (d) Construct an art gallery (polygon) that requires |n/3| guards."

DISCUSSION OF THE SOLUTION IN THE EXERCISE CLASS ON 23.05.2013.



Figure 1: Simple Polygon