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Proposed solutions for sheet 8

Graphs and Algorithms

Exercise 1 (Petersen, minors and subdivisions)

a) Remove an arbitrary vertex. For all vertices of degree two in the resulting graph combine the edges incident to them.

b) The Petersen graph has no vertices of degree 4.

c) The $K_{3,3}$ minor can be obtained in the same way as from a). The K_5 minor can be obtained by merging vertices on the inner star with their counterpart on the outer circle.

Exercise 2 (Outerplanar graphs)

First observe that a graph is outerplanar if and only if adding a vertex connected to all other vertices makes it a planar graph.

(a) Add a single vertex to the graph connected to all vertices, then run the planarity testing algorithm from the lecture.

(b) Add a single vertex to the graph connected to all vertices. Now since the graph is planar it is 4-colorable. The vertex we added has a unique color different from all other vertices. Removing it thus yields a three coloring of the original graph.

(c) Triangulate the art gallery and 3 color it. Put the guards on the color class of lowest cardinality, this yields the desired result.

(d) The following art gallery requires at least $n/3$ guards because each gray triangle can only be covered by one of the three vertices around it.