

Advanced Data Structures

Spring Semester 2018

Exercise Set 7

Exercise 1:

Consider k sorted arrays L_1, L_2, \dots, L_k , each of length n . We want to preprocess them, so that we can do binary search (predecessor or successor queries) in all of them simultaneously, with total cost $\mathcal{O}(k + \log n)$.

1. Sorting them leads to naive cost $\mathcal{O}(k \log n)$.
2. Consider new family of lists L' defined recursively as: L'_i is a sorted order of concatenation of L_i and $L'_{i+1}[2], L'_{i+1}[4], \dots, L'_{i+1}[n-2], L'_{i+1}[n]$ (all of L_i and every second element of L'_{i-1}).
3. Show that all lists L'_1, \dots, L'_k are still $\mathcal{O}(n)$ length.
4. Show that we can maintain pointers, so that given successor in L'_i , we can find successor in L'_{i-1} .
5. Show that we can maintain pointers. so that given successor in L'_i , we can find “true” successor (element of L_i) in $\mathcal{O}(1)$ time.

Now, we will step by step build a data structure for the 3D orthogonal range reporting ($[a_1, b_2] \times [a_2, b_2] \times [a_3, b_3]$) with $\mathcal{O}(n \log^3 n)$ space and the optimal $\mathcal{O}(\log n + k)$ query time, where k is the number of reported points.

Exercise 2:

In the Exercise Set 6, we already built a data structure for the dominance query ($[-\infty, b_2] \times [-\infty, b_3]$). Here, we discuss another data structure which is applicable in the 3D orthogonal range reporting. The rough idea is as follows:

1. In addition to the vertical rays, extend a horizontal line from each point until the endpoints hit another ray as shown in the left of Fig. 1. (It could be a line segment, a ray or a line.)
2. For a query point p , find out all the regions which are left to p and intersected by the y -coordinate of p , and report their right vertical rays as shown in the right of Fig. 1.

Please explain how to process this planar subdivision such that the query can be performed in $\mathcal{O}(\log n + k)$ time. The same idea also works for the $[a_2, \infty] \times [-\infty, b_3]$, $[a_2, \infty] \times [a_3, \infty]$, and $[-\infty, b_2] \times [a_3, \infty]$ queries.

Hint:

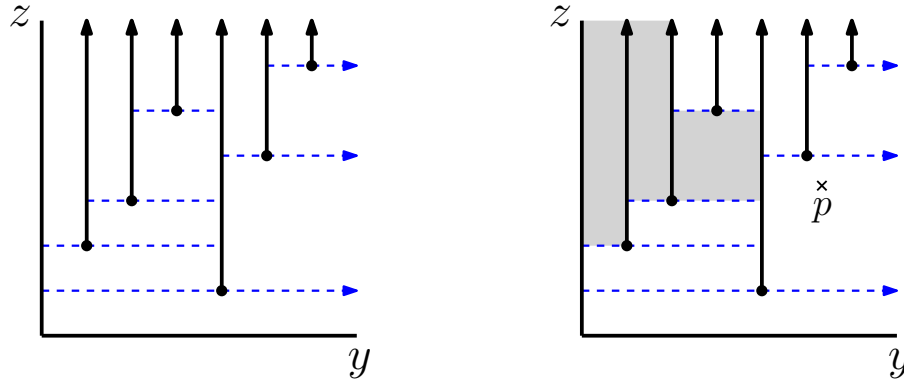


Figure 1: Left: A planar subdivision. Right: the query for p .

- A binary search from one region to another (from left to right) takes $\mathcal{O}(\log n)$ time, leading to $\mathcal{O}(\log^2 n + k)$ query time.
- Fractional cascading (Exercise 1) can improve the query time to $\mathcal{O}(\log n + k)$.

Exercise 3:

Build a data structure for the $[a_1, b_1] \times [-\infty, b_2] \times [-\infty, b_3]$ orthogonal reporting with $\mathcal{O}(n \log n)$ space and the optimal $\mathcal{O}(\log n + k)$ query time. The idea should also work for the $[a_1, b_1] \times [a_2, \infty] \times [-\infty, b_3]$, $[a_1, b_1] \times [a_2, \infty] \times [a_3, \infty]$, and $[a_1, b_1] \times [-\infty, b_2] \times [a_3, \infty]$ queries.

Hint: Build a range tree on the x -coordinates and apply Exercise 2 for the sub-tree of each node.

Exercise 4:

Build a data structure for the $[a_1, b_1] \times [a_2, b_2] \times [-\infty, b_3]$ orthogonal reporting with $\mathcal{O}(n \log^2 n)$ space and the optimal $\mathcal{O}(\log n + k)$ query time. The idea should also work for the $[a_1, b_1] \times [a_2, b_2] \times [a_3, \infty]$ query.

Hint:

- Build a range tree on the y -coordinates.
- For each node v of the range tree, apply Exercise 3:
 - to build a data structure for $[a_1, b_1] \times [-\infty, b_2] \times [-\infty, b_3]$ on the right tree of v ,
 - and to build a data structure for $[a_1, b_1] \times [a_2, \infty] \times [-\infty, b_3]$ on the left tree of v .

Exercise 5:

Build a data structure for the $[a_1, b_1] \times [a_2, b_2] \times [a_2, b_3]$ orthogonal reporting with $\mathcal{O}(n \log^3 n)$ space and the optimal $\mathcal{O}(\log n + k)$ query time.

Hint:

- Build a range tree on the z -coordinates.
- For each node v of the range tree, apply Exercise 4
 - to build a data structure for $[a_1, b_1] \times [a_2, b_2] \times [-\infty, b_3]$ on the right tree of v ,
 - and to build a data structure for $[a_1, b_1] \times [a_2, b_2] \times [a_3, \infty]$ on the left tree of v .