

Advanced Data Structures

Spring Semester 2018

Exercise Set 09

Below, \circ denotes concatenation.

Exercise 1:

Let our input word be $x = 0^t \circ \varepsilon_1 \circ 0^t \circ \varepsilon_2 \circ \dots \circ 0^t \circ \varepsilon_k$, where $\varepsilon_i \in \{0, 1\}$ (assume $t \gg \log w$). Show that there is constant m , such that we can extract $\text{bitcount}(x)$ from $(x \cdot m)$ using $\mathcal{O}(1)$ bit operations.

Exercise 2:

Parallel search in $\mathcal{O}(1)$ bit operations. That is, show that given $s_1 < s_2 < \dots < s_k$ of the same length, we can pack them into single word (assume you have enough space), that given q of the same length as s_i , we can find using constant number of operations, i such that $s_i \leq q < s_{i+1}$.

Hint: Assume you have some extra space (this is not a problem) and pack into single word $a_1 = 1 \circ s_1 \circ 1 \circ s_2 \circ \dots \circ 1 \circ s_k$ and consider $a_2 = 0 \circ q \circ 0 \circ q \circ \dots \circ 0 \circ q$. Use Exercise 1 on $a_1 - a_2$.

Exercise 3:

Most significant '1' on short inputs: given x such that $|x| = \sqrt{w}$, show that we can find position of the most significant '1' in x .

Hint: Use parallel search of x among \sqrt{w} words of length \sqrt{w} .

Exercise 4:

Most significant '1' on inputs as in Exercise 1: $x = 0^k \circ \varepsilon_1 \circ 0^k \circ \varepsilon_2 \circ \dots \circ 0^k \circ \varepsilon_k$ where $k = \sqrt{w}$.

Hint: There is explicitly given constant m , such that $(x \cdot m)$ contains a single block of bits: $\varepsilon_1 \varepsilon_2 \dots \varepsilon_k$.

Exercise 5:

Most significant '1' on any input.

Hint: Cut your input into \sqrt{w} blocks of length \sqrt{w} (mentally, we cannot afford it). With little work, we can find first non-empty block (and reduce that part to Exercise 4). Inside first non-empty block, use Exercise 3.