ETH Zurich
Institute of Theoretical Computer Science
Prof. Dr. Johannes Lengler
Konstantinos Lakis

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Handout 1

Complex Network Models

You may hand in your solutions (handwritten or typeset in LaTeX) to konstantinos.lakis@inf.ethz.ch if you want to get feedback

Exercise 1

Let us consider an undirected graph. We look at the following instance of Bidirectional BFS (BBFS): we fix two distinct vertices u and v, and run two parallel BFS searches starting at u and v. More precisely, at each step of the algorithm, we select **arbitrarily** one of the two sides and do one BFS step on that side. We stop when a vertex w has been discovered by the two searches. We found a path from u to v through w.

Algorithm 1 Bidirectional BFS

Require: Connected graph $G = (V, E)$ and u and v two vertices

```
function BBFS(u, v)
    Q_0 ← queue({u})
    Q_1 ← queue({v})
    D_0 ← set({u})
    D_1 ← set({v})
    V_0 ← set({})
    V_1 ← set({})
    mind, s ← ∞, None
    while True do
        x ∈ {0, 1}                                        ▷ arbitrarily
        w ← Q_x.pop()
        if w in V_x then
            continue
        V_x.insert(w)
        if w in V_{1−x} then
            return path u → s joint to path s → v
        for all n in neighbours of w do
            if n not in D_x a then
                D_x.insert(n)
                if n in D_{1−x} and we discovered a shorter path then
                    Update mind
                    s ← n
                Q_x.push(n)
```

1. Show that the path found is a shortest path between u and v.
2. Give an instance where BBFS runs faster than BFS from u.
3. Give an instance where BFS from u runs faster than BBFS.

Exercise 2

Let $n \in \mathbb{N}$, $p \in [0,1]$, and $G \sim G_{n,p}$. Let $T(G)$ be the random variable counting the number of (possibly overlaying) triangles in G. Compute $\mathbb{E}[T]$.

Exercise 3

We recall that for $\mu > 0$, $X \sim \mathrm{Po}(\mu)$ if and only if

$$\forall k \in \mathbb{N}_0, \mathbb{P}[X = k] = e^{-\mu} \frac{\mu^k}{k!}$$

1. Show that if $X \sim \mathrm{Po}(\mu)$, $Y \sim \mathrm{Po}(\lambda)$ are independent, then $X + Y$ is also Poisson-distributed.
2. Show that if (X_i) are independent Bernoulli of parameter $p \in [0,1]$ and $N \sim \mathrm{Po}(\mu)$ is independent from the (X_i), then $\sum_{i=1}^{N} X_i$ is also Poisson-distributed.