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Handout 10

Complex Network Models

You may hand in your solutions (handwritten or typeset in LaTeX) to konstantinos.lakis@inf.ethz.ch if you want to get feedback

Exercise 1

We consider a slight variation on the GIRG model. Instead of adding an edge $\{u, v\} \in \binom{V}{2}$ with probability $p(u, v) = \min\left\{1, \frac{w_u w_v}{||x_u - x_v||_\infty^d}\right\}^\alpha$, we instead use a step function $p(u, v) = \mathbb{1}\left\{\frac{w_u w_v}{||x_u - x_v||_\infty^d} \geqslant 1\right\}$. Show that we still have:

$$\mathbb{P}[u \sim v \mid w_u, w_v, x_u] = \Theta\left(\min\left\{1, \frac{w_u w_v}{n}\right\}\right).$$

Exercise 2

In a large enough GIRG,[1] consider the cubes $C_1 = -x + [-L/2, L/2]^d$ and $C_2 = x + [-L/2, L/2]^d$ for $x = (L, 0, 0, \ldots, 0) \in R^d$, i.e. the axis-aligned cubes of volume L^d centered at x and $-x$ respectively. We assume that $L^d = \Theta(n)$.

We are interested in $\mathbb{E}[X]$, where X is the random variable counting the number of edges (u, v) with $u \in C_1$ and $v \in C_2$. Recall that the connection probability is:

$$p_{uv} = \min\left\{1, \frac{w_u w_v}{|u - v|^d}\right\}^\alpha.$$

For the tasks below, you can focus on cases of the form $\tau > x$ or $\tau < x$ and do not need to consider equality (similarly for α).[2]

1. Compute $\mathbb{E}[X]$ up to constant factors.
2. Characterize the values of α and τ for which w.h.p. $X \geq 1$.

[1] So that the cubes are inside the geometric space of the GIRG.
[2] Where x does not even need to be a fixed number, but could be a function of α, τ.