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Handout 11

Complex Network Models

You may hand in your solutions (handwritten or typeset in LaTeX) to konstantinos.lakis@inf.ethz.ch if you want to get feedback

Exercise 1

Let X and Y be two independent random variables, distributed according to a strong density power-law of parameter $\tau > 1$. Show that XY has the following density:

$$\mathbb{P}[XY = x] = \Theta(x^{-\tau} \log x)$$

You can use that $\mathbb{P}[XY = x] = \int_1^x \frac{\mathbb{P}[X=u]\mathbb{P}[Y=x/u]}{u} du$

Exercise 2

Let G be a GIRG on the d-dimensional cube $\mathcal{X}$ with power-law exponent $2 < \tau < 3$. Let $1 \leqslant R \leqslant \frac{n^{1/d}}{4}$, and let $p \in \mathcal{X}$. Let $\mathcal{X}_s \subset \mathcal{X}' \subset \mathcal{X}_b \subset \mathcal{X}$ be the cubes centered in p and of radius respectively $\frac{R}{2}, R, 2R$. Let $V_s \subset V' \subset V_b \subset V$ be the vertices respectively contained in those cubes. Let $\mu = \min\{\alpha, \tau - 1\}$.

- Show that $\mathbb{E}[E(V_s, V_b \setminus V')] = \tilde{\Theta}(R^{d(2-\mu)})$
- Show that $\mathbb{E}[E(V', V_b \setminus V')] = \Omega(R^{d(1-1/d)})$