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Handout 2

Complex Network Models

You may hand in your solutions (handwritten or typeset in LaTeX) to konstantinos.lakis@inf.ethz.ch if you want to get feedback

Exercise 1

Following the definitions of Theorem 2.3, compute exactly $\mathbb{E}[n_3]$, p_3, $\mathbb{E}[n_4]$ and p_4.

Exercise 2

Theorem 2.4 shows that if $\mathcal{D} = \mathrm{Po}(\mu)$, with $\mu \neq 1$, then the probability to generate a Galton-Watson tree of size s is exponentially decaying in s. The assumption about the distribution is important. In this exercise, we show a weak counter-example. More precisely, find, for $\mu > 0$, $s \in \mathbb{N}$, a distribution $\mathcal{D}$ of mean μ such that $\mathbb{P}[|T| = s] \geqslant \frac{C}{s}$ for some C independent of s.

Hint: can you think of a distribution such that the tree consisting of a path of size $s-1$ is sampled with probability at least $\frac{C}{s}$ for some C independent of s?

Exercise 3

Let $n \in \mathbb{N}$, $k \in [n]$. Let U and V be sampled independently and uniformly at random among subsets of $[n]$ of size k. Find a threshold for the event that U and V intersect. More precisely, find a function $k_0 : n \mapsto k_0(n)$ such that :

1. $\mathbb{P}[U \cap V \neq \varnothing] = o(1)$ if $k(n) = o(k_0(n))$,
2. $\mathbb{P}[U \cap V \neq \varnothing] = 1 - o(1)$ if $k(n) = \omega(k_0(n))$.