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Handout 3

Complex Network Models

You may hand in your solutions (handwritten or typeset in LaTeX) to konstantinos.lakis@inf.ethz.ch if you want to get feedback

Exercise 1

Let $G \sim G_{n,p}$, with $p = \frac{\mu}{n}$, $\mu > 1$. One can show that G has a path of length $\Theta(n)$ w.h.p.. Using the sprinkling method, show that there exists a cycle of length $\Theta(n)$ w.h.p..

Exercise 2

For a graph G, we denote by N_d the number of vertices of degree d.

1. Let $G \sim G_{n,p}$, with $p = \mu/n$, $\mu > 0$. Show that $\mathbb{E}[N_1] = \Theta(n)$.
2. Let for $n \in \mathbb{N}$, $G(n) = ([n], E_n)$ be a graph sampled according to some distribution. We assume that
$$\forall d \in [n-1], \mathbb{E}[N_d] \geqslant \frac{n}{d^\tau}.$$
   (a) We assume $\tau < 2$. Show that $\mathbb{E}[|E_n|] = \Omega(n^{3-\tau})$.
   Hint: for $\alpha < 1$, $\sum_{k=1}^{n} \frac{1}{k^\alpha} = \Theta(n^{1-\alpha})$
   (b) Show that $\tau \geqslant 1$.
   (c) Show that if $\tau \leqslant 2$, then the graph is not sparse. More precisely, show that $\mathbb{E}[|E_n|] = \omega(n)$.

Exercise 3

Consider n i.i.d. random variables $X_1, \dots, X_n$ with distribution $\mathrm{Po}(\mu)$ for some constant μ. Let $X_{\max} := \max\{X_1, \dots, X_n\}$. Show that there exist constants C_1 and C_2 such that w.h.p. (i.e. with probability $1 - o(1)$ for $n \to \infty$):
$$C_1 \frac{\log n}{\log \log n} \leq X_{\max} \leq C_2 \frac{\log n}{\log \log n}$$

Hint: Throughout the course, you may use without proof the Stirling approximation:
$$n! = (1 \pm o(1))\sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$
or equivalently
$$\log(n!) = n \log n - n + \tfrac{1}{2}\log n + \tfrac{1}{2}\log(2\pi) \pm o(1)$$