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Handout 4

Complex Network Models

You may hand in your solutions (handwritten or typeset in LaTeX) to konstantinos.lakis@inf.ethz.ch if you want to get feedback

Exercise 1

Find a distribution $\mathcal{D}$ such that $\mathcal{D}$ follows a cumulative power-law with exponent $\tau > 1$, but such that $\mathcal{D}$ doesn't follow a weak power-law for any $\tau' > 1$.

Exercise 2

We recall the definitions. Let $c_1, c_2 > 0$ we say that a sequence $(u_k)_{k\in[n]} \in N^n$ follows

- a (c_1, c_2)-power-law distribution with exponent $\tau > 1$ and cut-off $D \in \mathbb{N}$, if

$$\forall d \in [D], c_1 \frac{n}{d^\tau} \leqslant \#\{k \in [n], u_k = d\} \leqslant c_2 \frac{n}{d^\tau}$$

- a (c_1, c_2)-cumulative power-law distribution with exponent $\tau > 1$ and cut-off $D \in \mathbb{N}$ if

$$\forall d \in [D], c_1 \frac{n}{d^{\tau-1}} \leqslant \#\{k \in [n], u_k \geqslant d\} \leqslant c_2 \frac{n}{d^{\tau-1}}$$

Let $\tau > 1$. Show that there exist $c_1, c_2 > 0$, such that, for all $n \in \mathbb{N}$, the sequence $u = \left(\left\lfloor \left(\frac{n}{i}\right)^{\frac{1}{\tau-1}} \right\rfloor\right)_{i\in[n]}$ follows

- a (c_1, c_2)-power-law distribution with exponent τ and cut-off $D(n)$,
- a (c_1, c_2)-cumulative power-law distribution with exponent τ and cut-off $D'(n)$.

What's the best cut-off you can achieve?