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## Complex Network Models

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*You may hand in your solutions (handwritten or typeset in LaTeX) to [konstantinos.lakis@inf.ethz.ch](mailto:konstantinos.lakis@inf.ethz.ch) if you want to get feedback*

### Exercise 1

In the following, we consider a graph  $G$  sampled according to a Chung-Lu model on  $n$  vertices with exponent  $\tau > 2$ . Let  $1 < w_0 < w_1$ , with  $w_1 > Cw_0$  for some  $C \geq 1$ .

1. We are interested in the probability that the weight of a fixed vertex is between  $w_0$  and  $w_1$ .
  - Give a condition on  $C$  such that this probability is  $\Theta(w_0^{1-\tau})$ .
  - What if the weights follow a cumulative power-law distribution of exponent  $\tau$ ? Give a condition on  $C$  under which we get the same result.
2. We assume  $w_1 \leq \frac{n}{w}$ . Let  $v$  be a vertex of weight  $w$ . What's the expected number of neighbours of  $v$  with degree between  $w_0$  and  $w_1$ ? Give the answer in  $\Theta$  notation under some suitable assumption on  $C$ .

### Exercise 2

In the following, we consider a graph  $G$  sampled according to a Chung-Lu model on  $n$  vertices with exponent  $\tau > 2$ . We sample an edge uniformly at random, and we take one of the two endpoints uniformly at random. Show that the distribution of the degree of that endpoint follows a strict power-law distribution up to a certain cutoff. Give the exponent and the cutoff.