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Handout 6

Complex Network Models

You may hand in your solutions (handwritten or typeset in LaTeX) to konstantinos.lakis@inf.ethz.ch if you want to get feedback

Exercise 1

Let G be sampled according to a Chung-Lu model on n vertices with exponent $2 < \tau < 3$. Let v be a vertex with weight $w_v \leqslant n^{\frac{\tau-2}{\tau-1}}$ such that $w_v = \omega(1)$. We are interested in the number of vertices $N_v^{(2)}$ at distance at most 2 from v. We recall that for all $\varepsilon > 0$, $w_v^{\frac{1}{\tau-2}-\varepsilon} \leqslant \max_{u\sim v} w_u \leqslant w_v^{\frac{1}{\tau-2}+\varepsilon}$ w.h.p.. Let $\varepsilon > 0$.

1. Show that w.h.p., $N_v^{(2)} \geqslant w_v^{\frac{1}{\tau-2}-\varepsilon}$
2. Show that w.h.p, $N_v^{(2)} \leqslant w_v^{\frac{1}{\tau-2}+\varepsilon}$
   Hint: Condition on $\max_{u\sim v} w_u \leqslant w_v^{\frac{1}{\tau-2}+\varepsilon}$

Exercise 2

We consider a Chung-Lu random graph on n vertices with exponent $\tau > 2$, what is the largest weight / degree that appears in the graph? Give a general formula, and show the following:

1. w.h.p., there is no weight larger than n in the graph.
2. If $\tau \in (2, 3)$ then w.h.p. there is a vertex of weight at least $\sqrt{n}$ in the graph.
3. If $\tau > 3$, then w.h.p. there is no vertex of weight at least $\sqrt{n}$ in the graph.

For each of the questions, is the error probability stretched exponentially small (i.e., of the form $e^{-n^{\Omega(1)}}$), or is it only polynomially small (of the form $n^{-\Theta(1)}$)?