ETH Zurich
Institute of Theoretical Computer Science
Prof. Dr. Johannes Lengler
Konstantinos Lakis

FS 2026
Handout 8

Complex Network Models

You may hand in your solutions (handwritten or typeset in LaTeX) to konstantinos.lakis@inf.ethz.ch if you want to get feedback

Exercise 1

We want to show that there is no efficient local routing (navigation) algorithm in Chung-Lu graphs.

Let G be a graph sampled according to a Chung-Lu model with exponent $\tau > 2$. We start from a vertex sampled uniformly at random, and the goal is to reach another vertex sampled uniformly at random. Initially, only the start vertex is uncovered. In each step, we may pick an uncovered vertex and *explore* it, meaning that we uncover its neighbours and their weights. Show that, whatever the strategy, the algorithm needs to uncover $\Omega(n)$ vertices in expectation in order to find the target vertex.

Hint: Assume that the target vertex has degree one, and has no neighbour of weight > 2*. What is the probability that this happens? If it happens, show that all vertices with weight in* $[1, 2]$ *have probability* $\Theta(1/n)$ *to be the unique neighbour. Deduce that in this case, the algorithm needs to explore* $\Omega(n)$ *vertices in expectation.*

Exercise 2

For a vertex v of degree ≥ 2, pick two different neighbours u_1, u_2 of v uniformly at random. The local *clustering coefficient* of v is the probability that u_1 and u_2 are adjacent.

The (averaged local) clustering coefficient of a graph G is the average clustering coefficient over all vertices of degree at least 2.[1]

Let G be a graph sampled according to a Chung-Lu model with strong density power law and with exponent $\tau < 3$. Let v a vertex of weight w_v. We want to provide an upper bound on the clustering coefficient of v. To do so, recall that vertices in the neighborhood of a vertex follow a strong density power law with exponent $\tau - 1$ up to some cutoff. For the upper bound, we will ignore the cutoff and just assume that the weights follow this distribution all the way to infinity. With this modification, what would the resulting upper bound on the clustering coefficient of v be?[2] What is then the upper bound for the clustering coefficient of G? Give the answer in Θ notation.

[1]There are different conventions on how to account for vertices of degree ≤ 1, but they will not affect the result.
[2]Where the expectation is over the random generation of G, except that the weight of v is fixed.