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Solutions to Handout 1

Complex Network Models

Solution to Exercise 1

1. In the following:
   - we mean by discovered from side x that it was put in D_x
   - we mean by visited from side x that it was put in V_x
   - we mean by the u-side (resp. the v-side) the BFS starting from u (resp. v), corresponding to $x = 0$ (resp. $x = 1$)

   Let w be the first vertex visited from both sides. Let d (resp. d') be the distance to w from u (resp. v). As w was visited from the u-side, all vertices at distance d from u were discovered from the u-side. Similarly, all vertices at distance d' from v were discovered from the v-side. The shortest path has distance at most $d + d'$. All such paths have one vertex which was discovered from the two sides. As we return the shortest distance path, we return the shortest path.

2. Choosing always the v side for BBFS will yield a much faster running time than a BFS starting from u.

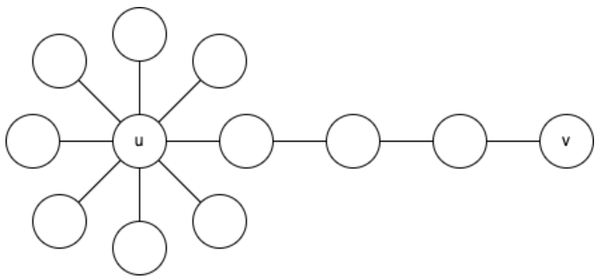

Figure 1: Graph

3. Choosing always the v side for BBFS will yield a much slower running time than a BFS starting from u.

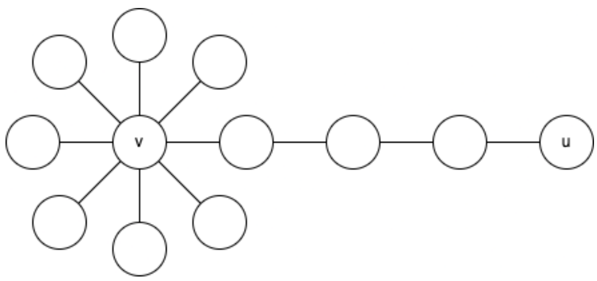

Figure 2: Graph

Solution to Exercise 2

Let for $t \in \binom{V}{3}$, X_t be the indicator random variable equal to one if and only if the triangle made by vertices of t is present. We have $T = \sum_{t \in \binom{V}{3}} X_t$, so by linearity of expectation:

$$\begin{aligned}
\mathbb{E}[T] &= \sum_{t \in \binom{V}{3}} \mathbb{E}[X_t] \\
&= \binom{n}{3} p^3
\end{aligned}$$

Solution to Exercise 3

1. Let $k \in \mathbb{N}_0$.

$$\begin{aligned}
\mathbb{P}[X+Y=k] &= \sum_{l=0}^{k} \mathbb{P}[X=l]\mathbb{P}[Y=k-l] && \text{by independence} \\
&= \sum_{l=0}^{k} e^{-\mu}\frac{\mu^l}{l!} e^{-\lambda}\frac{\lambda^{k-l}}{(k-l)!} \\
&= \frac{e^{-(\mu+\lambda)}}{k!} \sum_{l=0}^{k} \binom{k}{l} \mu^l \lambda^{k-l} \\
&= e^{-(\mu+\lambda)} \frac{(\mu+\lambda)^k}{k!}
\end{aligned}$$

2. Let $k \in \mathbb{N}_0$.

$$\begin{aligned}
\mathbb{P}\left[\sum_{i=0}^{N} X_i = k\right] &= \sum_{n \in \mathbb{N}_0} \mathbb{P}[N=n]\mathbb{P}\left[\sum_{i=0}^{n} X_i = k\right] && \text{by independence} \\
&= \sum_{n \geqslant k} e^{-\mu}\frac{\mu^n}{n!}\binom{n}{k} p^k (1-p)^{n-k} \\
&= e^{-\mu}\frac{(p\mu)^k}{k!} \sum_{n \geqslant k} \frac{1}{(n-k)!} (\mu(1-p))^{n-k} \\
&= e^{-\mu}\frac{(p\mu)^k}{k!} e^{\mu(1-p)} \\
&= e^{-p\mu}\frac{(p\mu)^k}{k!}
\end{aligned}$$