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Algorithmic Game Theory HS 2013

Exercise sheet 2
(This exercise is not compulsory but you will get feedback on submission.)

EXERCISE 2.1:
In the following games, find:

a) For every player, all pairs of strategies s, s' where s *weakly* dominates strategy s', and all pairs s, s' where s *strictly* dominates strategy s';

b) All Nash equilibria.

Are these games solvable by an iterative elimination of weakly dominated strategies? Are they solvable by an iterative elimination of strictly dominated strategies?

Game A. For an entry (i, j), i is the payoff of Rose and j is the payoff of Colin.

		Colin A	Colin B	Colin C
Rose	A	$(3,1)$	$(1,1)$	$(2,0)$
Rose	B	$(2,7)$	$(1,-1)$	$(1,2)$
Rose	C	$(1,-1)$	$(0,0)$	$(2,5)$

Game B. Here we have a strategic game with 3 players. The first table encodes the payoffs in case that Paul selects strategy A, while the second table is for Paul selecting strategy B. Paul's payoff is given by the third component of every payoff vector, Rose and Colin's as before.

Paul – A

		Colin A	Colin B
Rose	A	$(2,1,-1)$	$(1,0,2)$
Rose	B	$(0,7,0)$	$(1,9,1)$

Paul – B

		Colin A	Colin B
Rose	A	$(7,1,0)$	$(5,0,5)$
Rose	B	$(6,2,1)$	$(4,0,2)$

EXERCISE 2.2:
We define the following 2-player game:
A strategy s_i for a player $i = 1, 2$ is any value in the interval $[0, 1]$. Once both players have chosen a strategy, we declare winner the player whose strategy s_i is uniquely closest to $s = 0.9 \cdot \frac{s_1+s_2}{2}$. A player has utility 1 if he is the winner and utility 0 otherwise. Compute all the Nash equilibria of this game.

EXERCISE 2.3:
Consider a very oversimplified description of the Cuban missile crisis between the United States under John F. Kennedy and the Soviet Union under Nikita Khrushchev in 1963. Khrushchev starts the game by deciding whether or not to place intermediate range ballistic missiles in Cuba. If he does place the missiles, Kennedy has three options: do nothing, blockade Cuba, or eliminate the missiles by a surgical airstrike. If Kennedy chooses the aggressive action of a blockade or an airstrike, Khrushchev may acquiesce, or he may order escalation, leading to nuclear war. We do not give any explicit payoffs for the outcomes.

a) Draw the game tree corresponding to the described game.

b) Assume that the players in the Cuban missile crisis had the following preferences on the outcome of the game:
Kennedy:

   i) After the missiles are placed, Kennedy threatens with some aggressive action, after which Khrushchev backs down and acquiesces.
   ii) No missiles are placed on Cuba in the first place.
   iii) After the missiles are placed, Kennedy does nothing.
   iv) The crisis ends in a nuclear war.

   Khrushchev:

   i) After the missiles are placed, Kennedy does nothing.
   ii) No missiles are placed on Cuba in the first place.
   iii) After the missiles are placed, Kennedy threatens with some aggressive action, after which Khrushchev backs down and acquiesces.
   iv) The crisis ends in a nuclear war.

   Use backward induction on the game tree to find the rational outcome for this game. That is not what happened in 1963. Can you find some possible explanation for why not?

EXERCISE 2.4:
Consider an extensive game G given by a game tree T. A sub-game of G is every extensive game given by a sub-tree of T (where a sub-tree of T is a tree rooted in a vertex v of T and containing all descendents of v). Consider G as a strategic game (i.e., a game with the strategies S_i that prescribe the moves of player i in every node belonging to player i). A sub-game perfect Nash equilibrium of G is a strategy profile $s \in S = S_1 \times \ldots \times S_n$ which represents a Nash equilibrium in each sub-game of G. Prove the following Lemma that was already stated in the lecture:

> A solution to an extensive game that was derived using backward induction
> gives a sub-game perfect Nash equilibrium.

Hint: First prove that the solution is a Nash equilibrium, then that it is even a sub-game perfect Nash equilibrium.