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Algorithmic Game Theory HS 2013

Exercise sheet 9

EXERCISE 9.1:
Let $G = (V, E)$ be a 2-edge-connected graph, i.e., between any two nodes of the graph there exist at least two edge-disjoint paths.
Suppose that G is a network and that every edge (link) e is owned and operated by an agent A_e. A company is interested in buying a possibly cheap subset T of the links, such that every pair of nodes of G can communicate along edges of T. Hence, we assume that T induces a *spanning tree* of G. The company asks singularly every agent A_e for the cost t_e they incur in operating the link, which is a private information. The agent can possibly lie about the cost and report c_e to achieve a better utility, which is defined as $p_e - t_e$, if the link joins T and the agent is consequently awarded of p_e francs, or 0, otherwise.

Design a mechanism that induces the agents to truthfully announce their cost. Moreover, the agents utility should never be negative. The description of the mechanism reports how the links are selected and which payment the agents are awarded. Show that the mechanism is truthful.

EXERCISE 9.2:
Consider the mechanism design problem for the algorithmic problem of scheduling jobs (tasks) on related parallel machines as presented in the lecture. Recall that there are n machines (the players in the mechanism design setting), each having speed s_i with which it can process jobs. We denote by t_i the time which machine i needs to process a job of unit load, i.e. $t_i = \frac{1}{s_i}$. This denotes the cost a machine incurs when it processes a job of unit length. Here, t_i is the private information of the player. We are interested in truthful mechanisms that schedule m jobs with load $l_1, l_2, \ldots, l_m$ on the machines. A mechanism asks the machines to report their unit costs t_i (they can of course lie about it) and based on these reported values ("bids") $b = (b_1, b_2, \ldots, b_n)$ the mechanism assigns (somehow, according to the output function f) each job to exactly one machine and decides payments p_i for every machine i. By $W_i(b)$ (or W_i only, when the bids b are clear from the context) we denote the total load of jobs assigned to machine i when the players tell (bid) $b = (b_1, \ldots, b_n)$. We call W_i the work of machine i. The total cost of machine i is thus $W_i \cdot t_i$.

Show that the following social choice function f can be used to create a truthful mechanism for the considered mechanism design problem: Fix an arbitrary order of the n machines; Select (as $f(b_1, \ldots, b_n)$) the optimum job allocation (among all optimum job allocations) whose workload on machines, when seen as a vector $(W_1, W_2, \ldots, W_n)$, is lexicographically smallest.

EXERCISE 9.3:
In this exercise you are asked to prove a characterization of (any) truthful mechanism, which is stated in the following theorem.

Theorem. A mechanism is truthful if and only if it satisfies both the following conditions $\forall i, \forall v_{-i}$:

a) for any two v_i, v_i' that result in the same chosen outcome $a = f(v_i, v_{-i}) = f(v_i', v_{-i})$, the payment does only depend on the outcome. Formally, for every v_{-i}, for every $a \in A$, there exist prices $p_a \in \mathbb{R}$ such that for every v_i with $f(v_i, v_{-i}) = a$ the payment is $p_i(v_i, v_{-i}) = p_a$.

b) The mechanism optimizes for each player. Formally, for every v_i we have that

$$f(v_i, v_{-i}) = \arg\max_a (v_i(a) - p_a),$$

where the outcomes a are from the range of $f(\cdot, v_{-i})$.