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Datenstrukturen & Algorithmen Exercise Sheet 4 AS 16

Hand-in: Thursday, 20th October 2016 before the start of the lecture at 10:00 in the entrance area of ML D28. Please staple all sheets together and use this sheet as the cover page. Fill out the first two fields of the form below.

Exercise class (Room & Day): _____

Submitted by: _____

Corrected by: _____

Bonus points: ____

Exercise 4.1 *Various exercises.*

a) The algorithm of Karatsuba and Ofman for multiplying integer numbers calculates the product of two numbers recursively using a formula that contains – besides addition and multiplication with the base (in our case: 10) – three products. Give two numbers x and y, such that these three products are $(15 \cdot 26)$, $(11 \cdot 9)$, and $(15 - 11) \cdot (9 - 26)$.

$x =$ _____ $y =$ _____

b) Perform two iterations of *Insertion Sort* on the following array. The array has already been sorted by previous iterations up to the double bar.

2	5	9	15	10	1	6	11	12	8	7	20
1	2	3	4	5	6	7	8	9	10	11	12

1	2	3	4	5	6	7	8	9	10	11	12

1	2	3	4	5	6	7	8	9	10	11	12

Please turn over.

c) Consider the following recursive formula:

$$T(n) := \begin{cases} 9 + 4T(n/3) & n > 1 \\ 6 & n = 1 \end{cases}$$

Specify a closed and non-recursive form for $T(n)$ that is as simple as possible, and prove its correctness using mathematical induction.

Hint: You may assume that n is a power of 3.

Exercise 4.2 *Algorithm Design: Sums of Numbers.*

Let $A = (A[1], \ldots, A[n])$ be an Array of natural numbers that is sorted in ascending order.

a) Develop an algorithm with cost $\mathcal{O}(n \log n)$ that gets a natural number z as input and that decides, whether the array A contains two (not necessarily different) elements a and b, such that $a + b = z$.

b) Develop an algorithm that solves the problem of exercise part a) with cost $\mathcal{O}(n)$.