Eidgenössische
Technische Hochschule
Zürich

Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Federal Institute of Technology at Zurich

Department of Computer Science
Markus Püschel
Peter Widmayer
Thomas Tschager
Tobias Pröger
Tomáš Gavenčiak

15th December 2016

Data Structures & Algorithm — Solutions to Sheet P12 — AS 16

Solution P12.1 *Longest path in a directed acyclic graph.*

The solution was mostly outlined in the task description. The first step was to find a topological ordering using, for example, the following algorithm:

Initially, all the vertices are *unordered* and let O be the empty order. For every vertex v, we will keep the number of its unordered *in*-neighbors $d^-(v)$. Then set list L to be all the vertices with $d^-(v) = 0$.

Now take any vertex w from L, remove it from L, assign it to be next in order O, and then decrease $d^-(v)$ of all out-neighbors v of w by one (for ordering w). If $d^-(v)$ has become 0 for some v, add such v to L. Repeat until all vertices are in the order $O = (v_1, v_2, \ldots, v_n)$.

The next step is to compute the longest path. A crucial observation is that any directed paths moves forward in the order O above and if there is a directed edge $w \to v$, then any path ending in w may be extended to a one-longer path ending in v. Therefore, for every vertex v, we may directly compute the length $l(v)$ of the longest path ending in v if we know this information for all the vertices w with direct edge $w \to v$. This leads to a dynamic program computing $l(v_1), l(v_2), \ldots, l(v_n)$ in that order as

$$l(v_i) = \max_{v_j \text{ an in-neighbor of } v_i} l(v_j) + 1,$$

or $l(v_i) = 1$ if v_i has no in-neighbors.

Solution programs On the lecture website, you can find a the solution sources with further comments on the implementation.

Data Similarly to the `test*` cases, test cases `judge1` and `judge2` were random directed acyclic graphs with various density of edges. The longest path lengths for such graphs were in the range 6–50 (even for graphs with 2000 vertices and 20 000 directed edges). The data also contained some edgeless graphs, long simple directed paths, and tournament graphs (where every two vertices share a directed edge in one of the directions).